2015 ACCA Calculus Competition

Multiple Choice

1. If $f(0) = 4$, $f'(0) = 2$, $f'(3) = 1$, $f'(-5) = -3$, $g(0) = 3$, and $g'(0) = -5$, find $(fg)'(0)$

   (a) -10 (b) 26 (c) -5 (d) -3 (e) -14

2. Let f be the function defined for all $x > 0$ by $f(x) = (\sqrt{x})^x$. Which of the following statements is **false**?
   (a) $\lim_{x \to 0^+} f(x) = 1$
   (b) $\lim_{x \to \infty} f(x) = \infty$
   (c) $f(x) = x^{x/2}$ for all $x > 0$
   (d) The derivative $f'(x)$ is positive for all $x > 0$.
   (e) The derivative $f'(x)$ is increasing for all $x > 0$.

3. Let $x_1 = 1$ and $x_{n+1} = \sqrt{3 + 2x_n}$. If $\{x_n\}$ converges, then $\lim_{n \to \infty} x_n =$

   (a) -1 (b) 0 (c) $\sqrt{5}$ (d) e (e) 3

4. Solve the differential equation $\dfrac{dx}{dt} = 1 - t + x - xt$
   (a) $x(t) = C + te^t$
   (b) $x(t) = 1 + Ce^{t/2}$
   (c) $x(t) = 1 - Ce^{t^2/2}$
   (d) $x(t) = Ce^{t - t^2/2}$
   (e) $x(t) = -1 + Ce^{t - t^2/2}$

5. If $\mathcal{C}$ is the square with verticies $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$, oriented counterclockwise, find $\displaystyle\oint_{\mathcal{C}} (3y\,dx + 4x\,dy)$

   (a) 0 (b) 1 (c) 3 (d) 4 (e) 7

6. If $f(x) = \int_1^{x^2} \frac{1}{1+t^3}\,dt$ then $f'(2) =$

(a) $\frac{4}{65}$

(b) $\frac{1}{9}$

(c) $\ln\left(\frac{65}{2}\right)$

(d) $\ln\left(\frac{9}{2}\right)$

(e) 0.23

7. Find the volume of the solid of revolution obtained by rotating the region bounded by $y = x^2 + 4$ and $y = 12 - x^2$ about the line $y = -1$

(a) 384π

(b) 128π

(c) 128

(d) 96π

(e) None of these

8. If $f(x) = \left\{ \begin{array}{ll} \sqrt{1-x^2} & \text{for } 0 \leq x \leq 1 \\ x-1 & \text{for } 1 < x \leq 2 \end{array} \right\}$, then $\int_0^2 f(x)\,dx$ is

(a) $\frac{\pi}{2}$

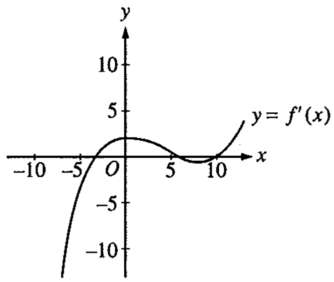
(b) $\frac{\sqrt{2}}{2}$

(c) $\frac{1}{2} + \frac{\pi}{4}$

(d) $\frac{1}{2} + \frac{\pi}{2}$

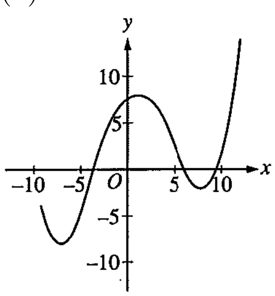
(e) undefined

9. The graph of $y = f'(x)$ is below

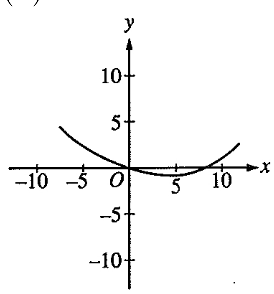


Which of the following could be the graph of $y = f(x)$?

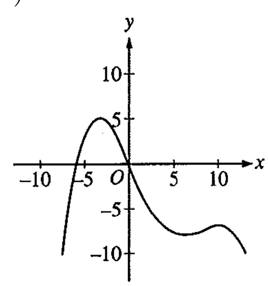
(a)



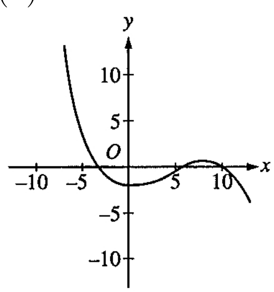
(b)



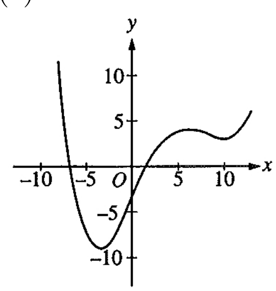
(c)



(d)



(e)



10. $\sum_{n=0}^{\infty}(2^{-n-2} - 3^{-n-2}) =$

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{1}{5}$

(d) $\frac{1}{6}$

(e) Does Not Exist

11. Evaluate the improper integral $\displaystyle\int_2^\infty \frac{dy}{y^2+2y-3}$

(a) $\frac{5}{2}$ (b) $\frac{1}{\ln 4}$ (c) $\frac{5}{4}$ (d) $\frac{\ln 5}{4}$ (e) divergent

12. If $\mathbf{a} = \langle 3, 5, 1\rangle$ and $\mathbf{b} = \langle -5, 2, -2\rangle$ find $\mathbf{a} \times \mathbf{b}$
    (a) $\langle -10, -5, 6\rangle$
    (b) $\langle -12, 1, 31\rangle$
    (c) $\langle 2, -6, -25\rangle$
    (d) $\langle -8, -11, -19\rangle$
    (e) $\langle -16, -23, 25\rangle$

13. An ice cube melts uniformly at a rate of $2\text{in}^3/\text{min}$ without changing its shape. How fast is the surface area changing when the volume of the ice cube is 27in^3?
    (a) $\frac{1}{9}\text{in}^2/\text{min}$
    (b) $-\frac{8}{3}\text{in}^2/\text{min}$
    (c) $1\text{in}^2/\text{min}$
    (d) $-4\text{in}^2/\text{min}$
    (e) None of these

14. Find the equation of the tangent plane to the surface $z = e^{-x}\sin y$ at $x = 0$ and $y = \frac{\pi}{2}$.
    (a) $x + y = 1$
    (b) $x + z = 1$
    (c) $x - z = 1$
    (d) $y + z = 1$
    (e) $y - z = 1$

15. $\displaystyle\int_0^1 \int_0^x xy\, dy\, dx =$

(a) 0 (b) $\frac{1}{8}$ (c) (d) 1 (e) 3

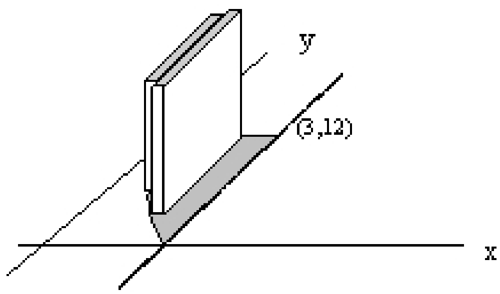
Short Answer

1. Find $\lim_{x \to 0} \dfrac{\sin 2x}{(1-x)\ln(1-x)}$

2. Find the maximum directional derivative at the point $P(0,1)$ on the surface $z = f(x,y) = xe^{xy} + y\cos x$

3. Given $p(x) = \sum_{k=1}^{\infty} \dfrac{(x-2)^k}{k^2}$, find the interval of convergence for $p'(x)$

4. The volume of the solid with the region bounded by $y = x^2 - 6x + 9$, $y = x + 9$, and $x = 3$ as its base and with square cross sections perpendicular to the base and x-axis is shown below. Find the volume of the solid.



5. Find the cubic equation $f(x) = ax^3 - bx^2 + cx - d$ that has a local maximum value of 60 at $x = 1$ and a local minimum of -4 at $x = 3$.

6. Find the equation of the tangent line to the curve $x\sin 2y = y\cos 2x$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{4}\right)$.

7. Evaluate $\displaystyle\int_0^{\frac{3}{2}} \int_{\sqrt{3}x}^{\sqrt{9-x^2}} e^{x^2+y^2}\, dy\, dx$

8. Find the absolute minimum value of $f(x,y) = 6 + 3xy - 2x - 4y$ on the region R bounded by the parabola $y = x^2$ and the line $y = 4$.

9. Let $f(x)$ be a function such that $f(x) = f(1-x)$ for all real numbers x. If f is differentiable everywhere, find $f'(0)$.

10. Find the length of the curve $x(t) = t\cos t$, $y(t) = t\sin t$ for $0 \le t \le 1$.